Spatial analysis and spatial dependence

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Overview

Spatial dependence and spatial statistics

- Some examples of spatial dependence
- Visualization of spatial data
- Specifying dependence structures
- Testing for spatial dependence
- Spatial autoregressive linear model
- Pointers to advanced topics

Examples of spatial dependence

Tale of two neighbors, Promethia and Tragedia

- Promethia does everything right, but below average growth rate
- Neighbor Tragedia has bad policies and disastrous economic performance
- Promethia's low growth rate reflects spill-over effects from Tragedia
- Observations for Promethia and Tragedia clearly not independent
- Easterly & Levine find that Africa term in cross country growth models becomes non-significant when controlling for contagion

Examples of spatial dependence

Civil wars not determined by attributes of states alone, events in other states can influence risk

- Central Africa: Uganda, Rwanda, Zaire, Kashmir conflict

Local character of environmental problems

- Murdoch and Sandler: SO_2 emission reductions in Europe easier to achieve since states more private benefits from reducing omissions
- Less local problems (e.g., NOX and $CO2$) more difficult

Visualization

Visualization may suggest important structure in data

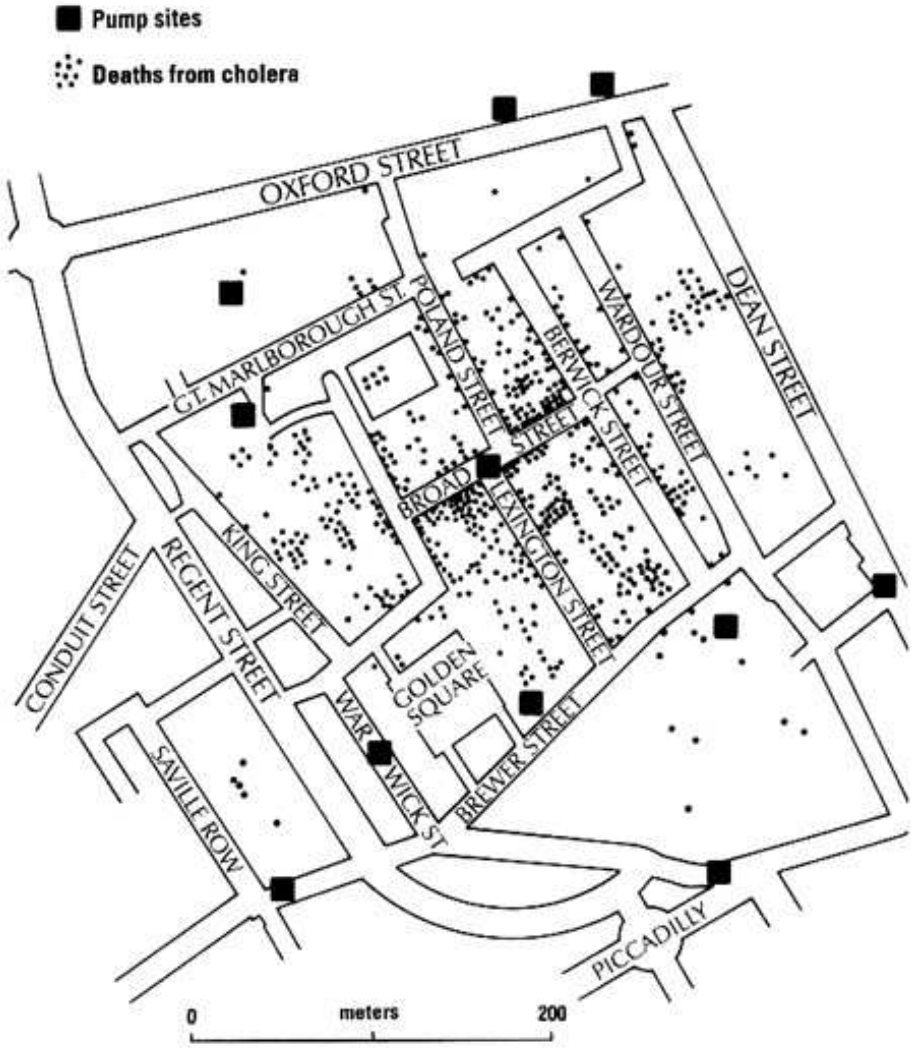
- Tables large, often unwieldy
- Maps can summarize information on one page
- Where are the high and low values located?
- Spatial structure/pattern?

The Snow map

Snow's map of cholera deaths in London

- September 1854 cholera epidemic
- 500 from same section of London died within a ten-day period
- Bacteria not yet know, sources of disease obscure
- Snow had previously published paper on water as possible source of spread of cholera
- Created a map of location of deaths and water pump

The Snow map



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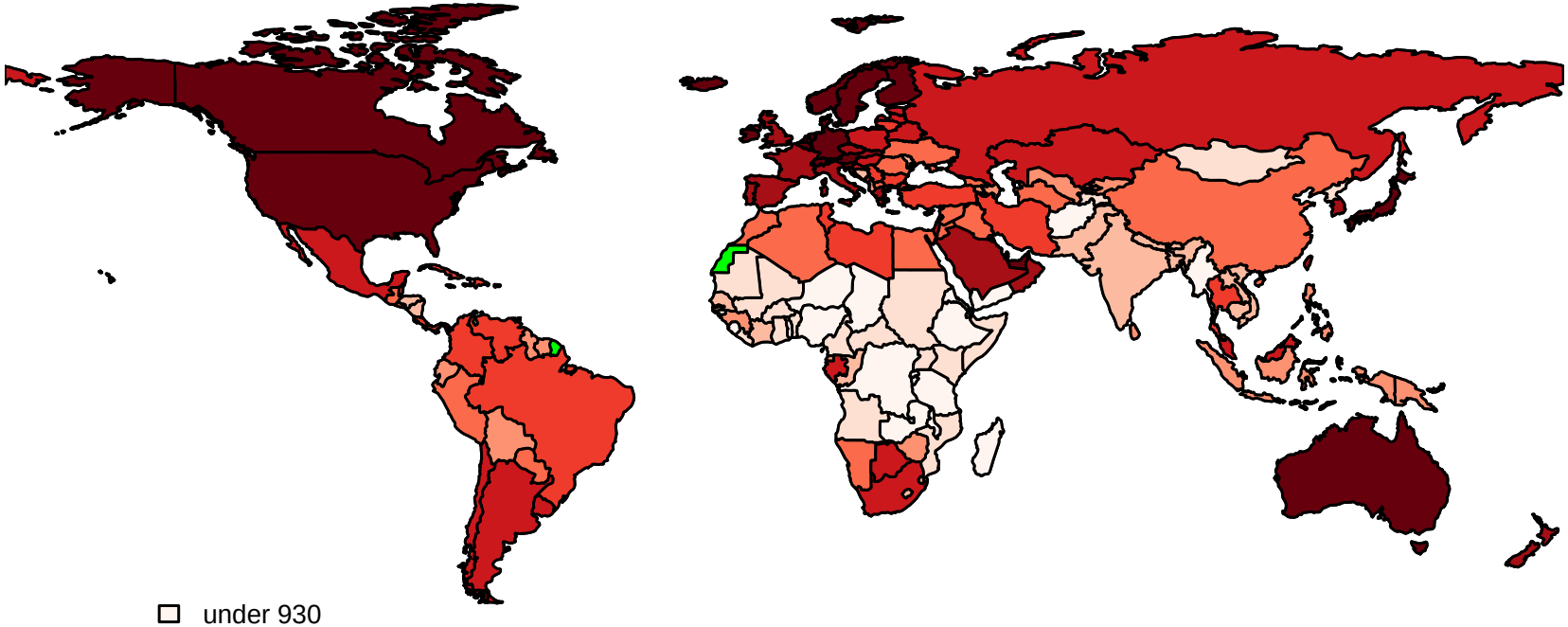
Global income distribution



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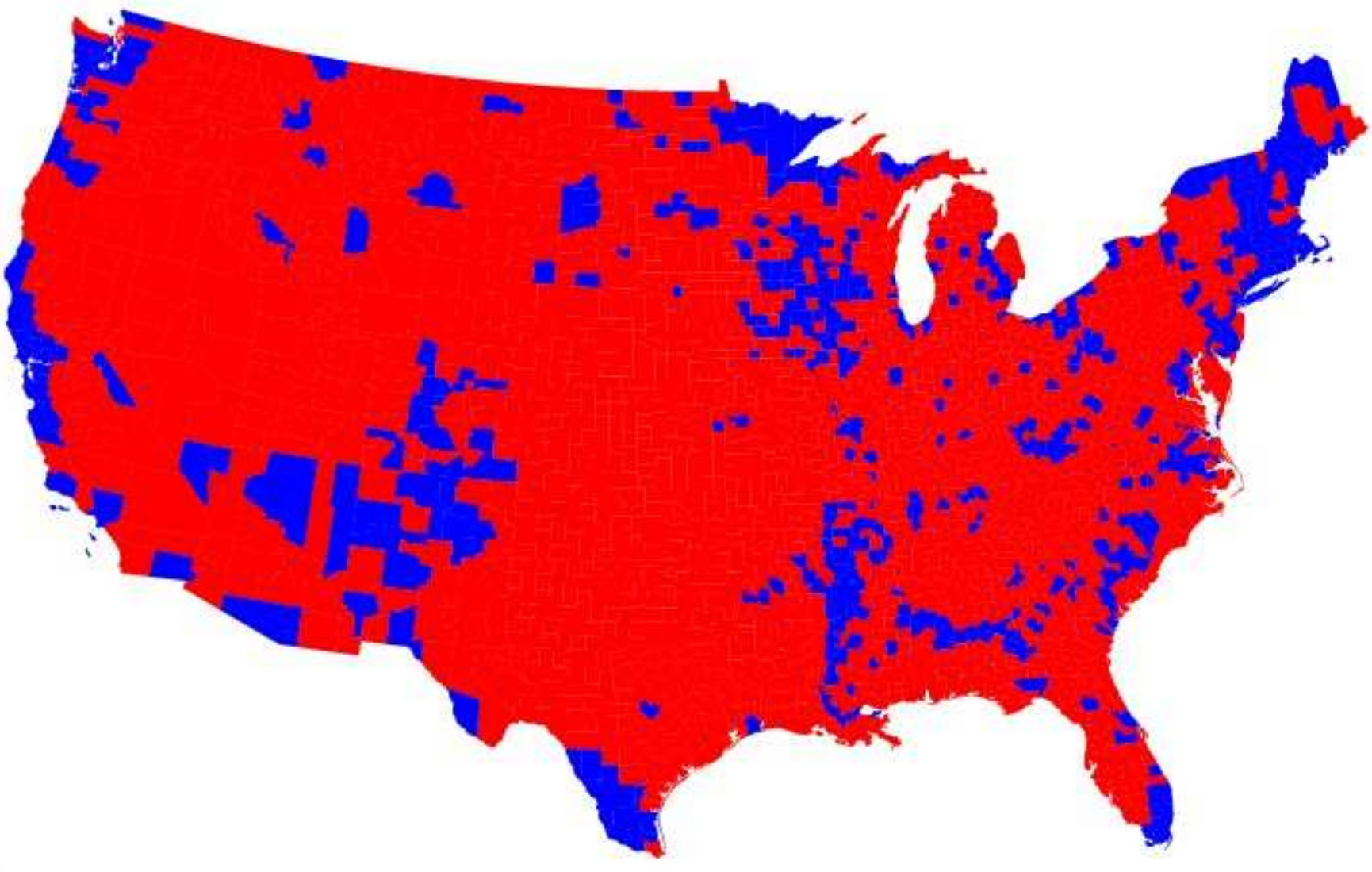
Red (Bush) and blue (Kerry) counties



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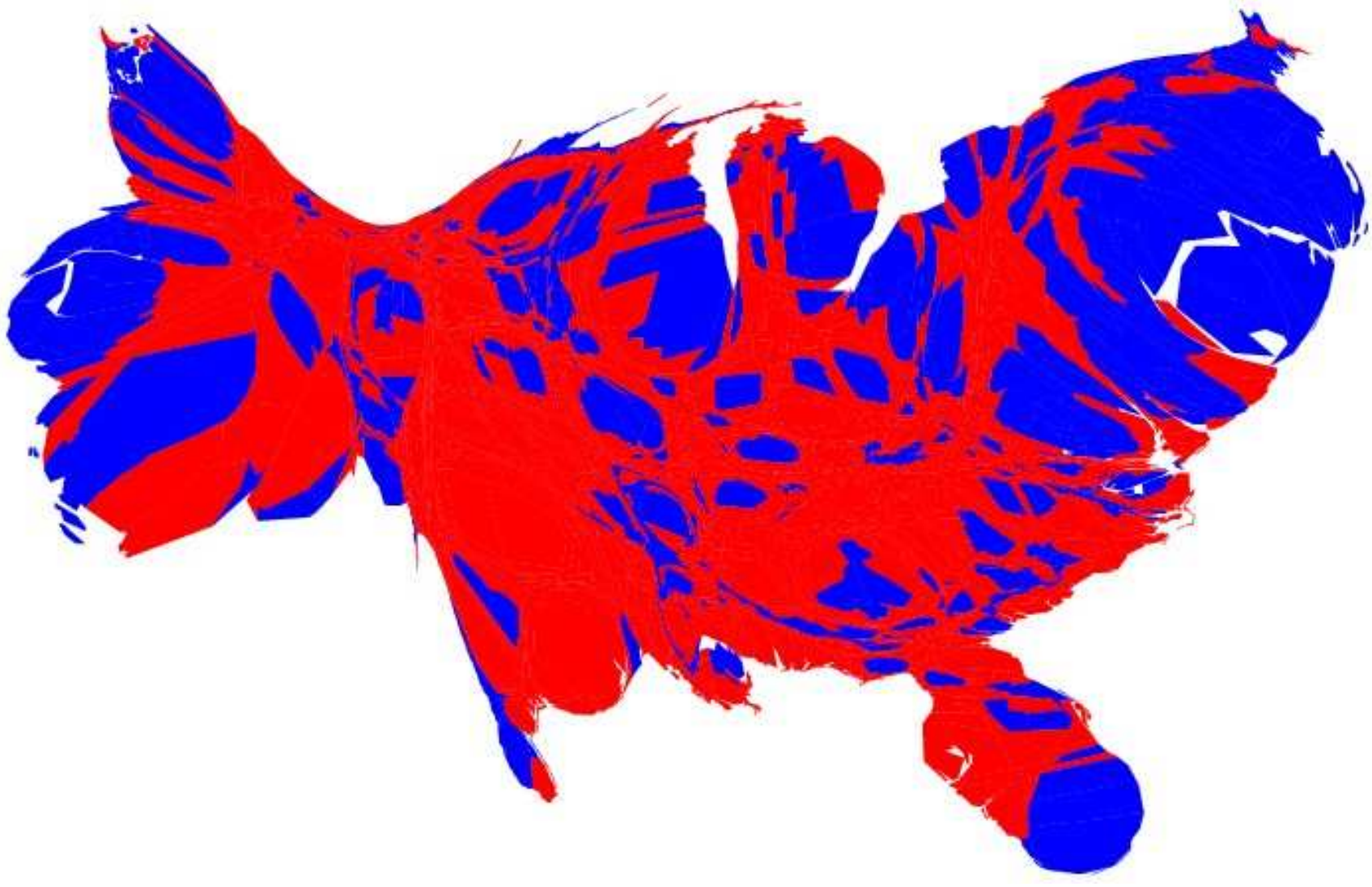
Political maps and cartograms

Displaying geographical shapes/units alone can be misleading

- Counties have very different population size
- More rural than urban counties, associated with differences in voting patters
- Map of blue and red populations should weight by population
- Cartogram scaling each county by population size

Red and blue counties revisited



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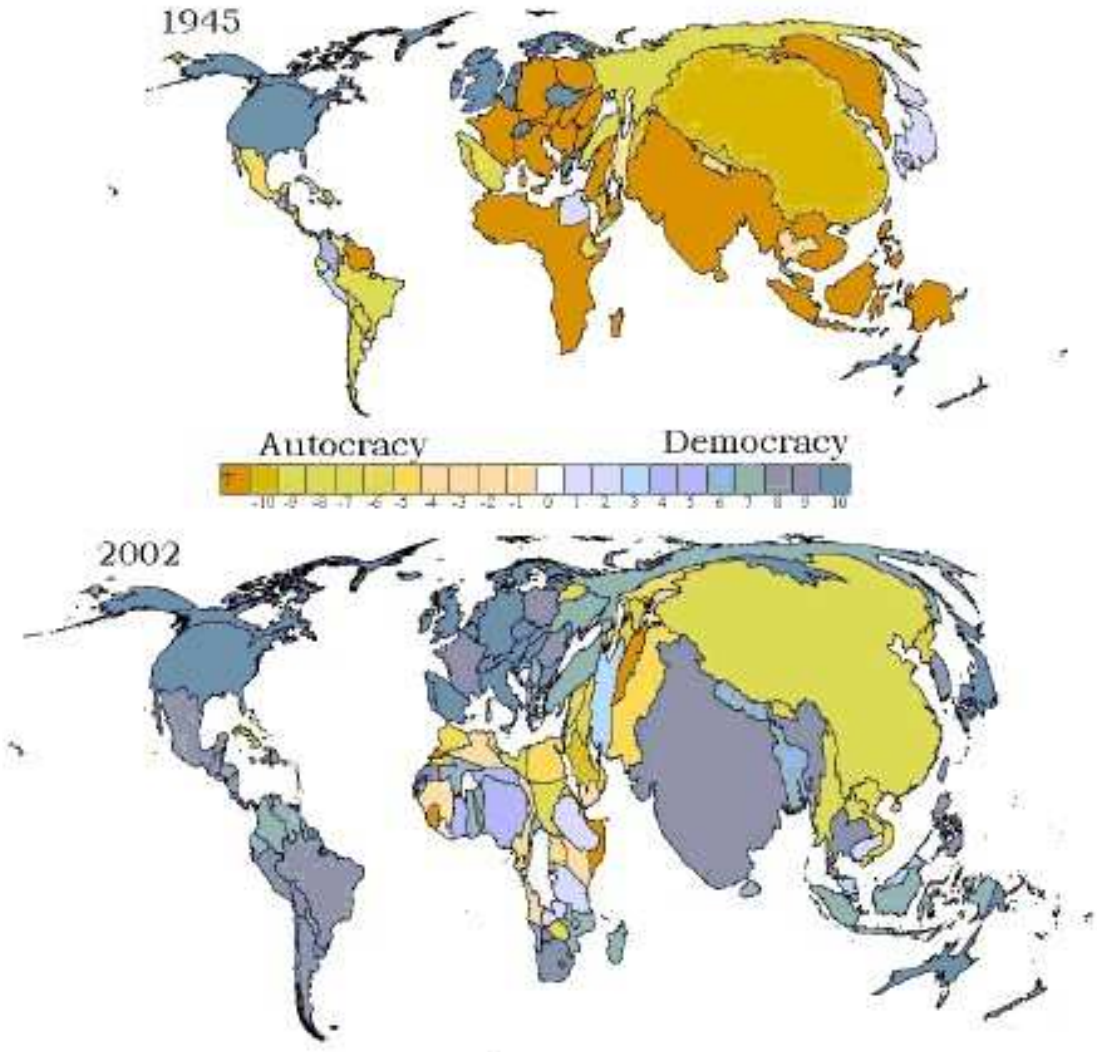
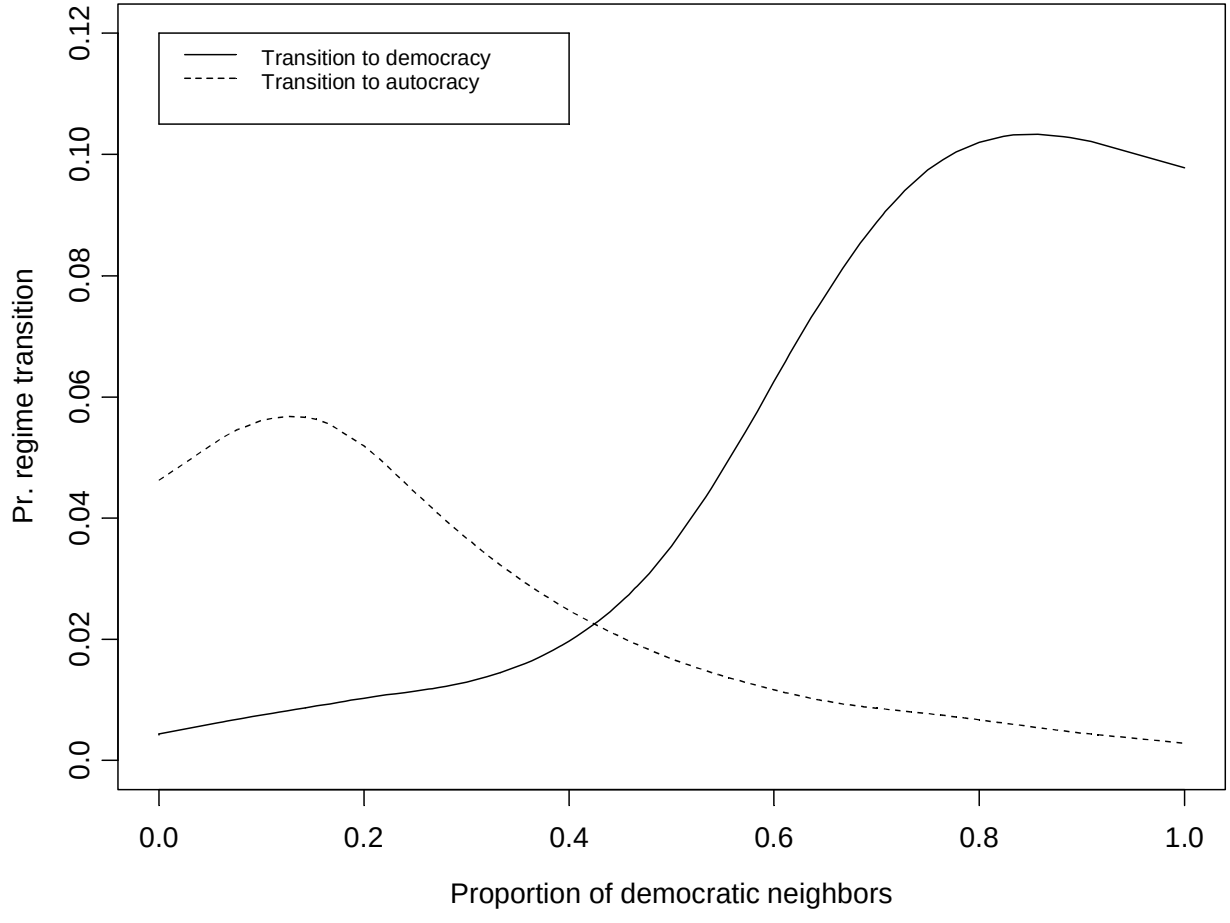
Democratization in the World



Democratization in the World



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Mapping data

- Most software proprietary and expensive (*ESRI ArcView, ArcInfo*)
- However, possible to use *ESRI* shapefiles to generate maps in the free package *R*
- `http://www.r-project.org`
- Shapefiles for many areas freely available on web
- For utilities to create diffusion based cartograms, see `http://www-personal.umich.edu/~mgastner/`

Spatial dependence

- Values y_i and y_j often similar for neighbors i and j
- Many diagnostic tests for spatial dependence available
- Global dependence: Moran's I
- Tests for spatial dependence presumes a pre-specified form of dependence
- Hence, tests only able to reject/not reject particular specification

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Connectivity

Pattern of connectivities between observations

- Connectivity matrix $\mathbf{W}$
  - $N \times N$ matrix where entries $w_{i,j}$ acquire non-zero values if units i and j connected
  - Alternatively, inverse distance weights based on distance d_{ij} where $w_{ij} = \frac{1/d_{ij}}{\sum_{j=1}^{n} 1/d_{ij}}$
- Criteria for connectivity?
- Time has inherent ordering, but $N \times (N-1)$ possible relationships in space for N units

Connectivity

Many possible forms of distance/connectivities

- Geography-distance link often suggests geographical dependence (e.g., contiguous states, proximate states)
- Observed interaction patterns (e.g., trade, communications)
- Cultural distance (e.g., links between languages)
- Reachability indices vs. geographical distance (e.g., time to travel between points)

Must be justified in each application

Moran's I

How similar is i to its neighbors J?

$$I = \frac{N \sum_i \sum_j c_{ij} (y_i - \bar{y}) (y_j - \bar{y})}{\left(\sum_i \sum_j c_{ij} \right) \sum_i (y_i - \bar{y})^2} \quad (1)$$

where c_{ij} is an element in a binary connectivity matrix $\mathbf{C}$

$Z = I/SE(I)$ indicates whether clustering is "significant"

E.g., Moran's $I = 0.47$,$Z > 10$ for spatial clustering in democracy, reject $H0$ of no clustering

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Spatial dependence

Usually assume that dv $Y = f(X)$

- Looking at unconditional spatial dependence problematic since the ivs X also likely to be non-randomly distributed
- Test whether residuals from a conditional model displays residual spatial clustering
- Ex: residuals from regression of expected democracy conditional on development still shows spatial dependence
- Moran's $I = 0.40, Z > 8$ for spatial clustering in residuals from regression, reject $H0$ of no clustering, even after controlling for GDP per capita

Spatial dependence

Why is spatial dependence a problem in a regression model?

- Regression assumptions imply that errors of individual observations should be independent and unrelated: $E[\epsilon_i, \epsilon_j] = 0$ for $i \neq j$.
- Spatial dependence violates this assumption, since $Cov(y_i, y_j)$ tends to be positive
- Standard properties do not hold
- More fundamentally, a model assuming that observations are independent is fundamentally incorrect

Spatial regression models

Introduce spatial structure into regression model

Spatial autoregressive (SAR) model: add a right hand side "spatial lag" $\mathbf{W}Y$, where $\mathbf{W}$ is row-normalized

$$E(y_i|x_i, Y) = \widehat{\beta}_0 + \widehat{\beta}_1 x_1 + \widehat{\rho} w_i.Y, \tag{2}$$

Estimation more difficult since y now on both sides of

equation, but can be estimated by MLE or IV

Spatial regression models

	(1)	(2)	(3)
Variable	OLS	Spatial autoregressive estimates	
Constant	-19.71 (3.66)	-12.96 (3.35)	-19.39 (3.39)
Ln(GDPPC)	2.66 (0.42)	1.72 (0.41)	2.22 (0.41)
ρ_1 (distance)		0.48 (0.09)	
ρ_2 (trade)			0.51 (0.14)
N	170	170	170

SAR results

Interpretation of SAR

- ρ estimate of spatial dependence, influence of neighbors
- Model implies feedback:
  - Change in IV in one state will first affects its DV
  - Then affect other states
  - Feed back to country itself
- Hence, coefficients for ivs $\mathbf{X}$ not fully comparable to OLS, must consider implied “equilibrium effect”

SAR results

$$
\begin{aligned}
Y &= \rho \mathbf{W} Y + \mathbf{X}\beta + \epsilon \\
\epsilon &= (I - \rho \mathbf{W}) Y - \mathbf{X}\beta \\
Y &= (I - \rho \mathbf{W})^{-1} \mathbf{X}\beta + (I - \rho \mathbf{W})^{-1} \epsilon \\
E(Y) &= (I - \rho \mathbf{W})^{-1} \mathbf{X}\beta
\end{aligned}
$$

I.e., the “equilibrium effect” of a change in IV will depend upon connectivities with other states, and vary from country to country

SAR results

SAR with geographical distance:

- effect of a unit increase in log of GDP per capita ranges from a 1.75 to a 2.04 point increase in level of democracy, average effect 1.80

SAR with trade connectivities:

- a one unit change in ln GDP per capita on average leads to a 2.24 point increase in democracy, effects for individual countries ranging from 2.22 to 2.52

SAR with two matrices

$$y_i = \mathbf{x}_i\beta + \rho_1 \mathbf{w}_i^A \mathbf{y} + \rho_2 \mathbf{w}_i^B \mathbf{y} + \varepsilon_i$$

SAR with two matrices

Table 1: Democracy and Social Requisites, 1998

	(1)	(2)	(3)	(4)
Variable	OLS	Spatial autoregressive estimates		
Constant	-19.71 (3.66)	-12.96 (3.35)	-19.39 (3.39)	-13.24 (3.11)
Ln(GDPPC)	2.66 (0.42)	1.72 (0.41)	2.22 (0.41)	1.53 (0.37)
ρ_1 (distance)		0.48 (0.09)		0.89 (0.19)
ρ_2 (trade)			0.51 (0.14)	0.59 (0.43)
N	170	170	170	170

Binary dependent variables

- SAR less appropriate for categorical dv's such as conflict, autologistic possible alternative
- Assume a locally dependent Markov field
  - $Pr(y_i \mid y_j, j \neq i)$ depends only on $y_j \iff j$ is a neighbor of i or $w_{i,j} = 1$

In the autologistic, condition likelihood of $y_i = 1$ on conflict among neighbors $\widetilde{y_i}$

$$Pr[y_i = 1|\widetilde{y_i}] = \frac{e^{\alpha + \mathbf{X}_i' \beta_k + \gamma \widetilde{y_i}}}{1 + e^{\alpha + \mathbf{X}_i' \beta_k + \gamma \widetilde{y_i}}}$$

- If $\gamma = 0$ this is a standard logistic, observations not conditional on one another

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Example of autologistic

Ward and Gleditsch (2002), N=138

			Spatial Parameters	
	Intercept	Democracy	Democracy	Conflict
Estimator	α	β_1	β_2	γ
Logistic	*-1.309*	-0.022	-0.015	–
MPL $\hat{\psi}$	*-1.840*	-0.020	0.013	*0.298*
MCMC ML $\hat{\theta}$	*-1.712*	*-0.053*	-0.003	*0.261*
Logistic SEs	0.218	0.033	0.048	–
MPL SEs	0.333	0.033	0.051	0.126
MCMC SEs	0.060	0.006	0.010	0.013

Example of autologistic

Prediction based on 1988 estimates for 1989 to 1998 period, $\hat{\pi}$ =0.35 threshold

	Observed	
Predicted	No	Yes
No	60	17
Yes	29	33